

# Pset 3: SVMs

$\circ$  hinge loss =  $\max(0, 1 - y_i \cdot (x_i \cdot w))$   
 ↑ true label    ↑ i-th row of input    ↑ weight vector  
 $-1$  or  $+1$      $\underbrace{\hspace{10em}}$  Score = data · weight vector for i-th  
 regularization term  $\lambda$  penalizes large weights

$\circ$   $w^* = \underset{w \in \mathbb{R}^d}{\operatorname{argmin}} \frac{\lambda}{2} \|w\|^2 + \frac{1}{N} \sum_{i=1}^N [1 - y_i (w \cdot x_i)]_+$   
 where  $(x_1, y_1) \dots (x_N, y_N) \in \mathbb{R}^d \times \{\pm 1\}$      $[.]_+ = \max(0, \cdot)$

$\frac{\partial}{\partial w} \max(0, \text{margin}) = -y \cdot x$

$\circ$  gradient  $\begin{cases} -y \cdot x & \text{if } 1 - y \cdot \text{score} > 0 \\ 0 & \text{if } 1 - y \cdot \text{score} \leq 0 \end{cases}$   
 " " +  $\lambda \cdot w$  if " " flat for margins  $\leq 0$   
 " " +  $\lambda \cdot w$  if " "   
 when you add regularization

miscategorized or within margin boundary  
 " correctly classified and outside margin boundary (not support vectors)

Gaussian / RBF Kernel:

$\circ$   $K(x, y) = \exp(-\gamma \|x - y\|^2)$   
 ↑ kernel SVM maintains  $K$  support vectors and learning weights  $\alpha_1, \dots, \alpha_K \geq 0$   
 ↑ positive constant  $\gamma = \frac{1}{2\sigma^2}$  variance  
 ↑  $\exp(x) = e^x$   
 ↑ Euclidean distance between each pair of points.

$X = \begin{matrix} N \\ \left[ \begin{matrix} & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \end{matrix} \right] \\ d \end{matrix}$      $K = \begin{matrix} M \\ \left[ \begin{matrix} & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \end{matrix} \right] \\ M \end{matrix}$   
 each entry  $i, j$  contains kernel function result using  $X[i]$  and  $Y[j]$

number of support vectors or can be # of data points to compare with  $X$  matrix

- $\circ$  learning rate is the amount the weights are updated between 0 and 1, also called step size
- $\circ$  pegasos algorithm penalizes primal objective with

